

ЧАСТЬ 2. РЯДЫ

Задача 1. Найти сумму ряда.

1. $\sum_{n=9}^{\infty} \frac{2}{n^2 - 14n + 48}.$

2. $\sum_{n=9}^{\infty} \frac{18}{n^2 - 13n + 40}.$

3. $\sum_{n=8}^{\infty} \frac{4}{n^2 - 12n + 35}.$

4. $\sum_{n=8}^{\infty} \frac{36}{n^2 - 11n + 28}.$

5. $\sum_{n=7}^{\infty} \frac{6}{n^2 - 10n + 24}.$

6. $\sum_{n=7}^{\infty} \frac{54}{n^2 - 9n + 18}.$

7. $\sum_{n=6}^{\infty} \frac{8}{n^2 - 8n + 15}.$

8. $\sum_{n=6}^{\infty} \frac{72}{n^2 - 7n + 10}.$

9. $\sum_{n=5}^{\infty} \frac{10}{n^2 - 6n + 8}.$

10. $\sum_{n=5}^{\infty} \frac{90}{n^2 - 5n + 4}.$

11. $\sum_{n=4}^{\infty} \frac{12}{n^2 - 4n + 3}.$

12. $\sum_{n=4}^{\infty} \frac{18}{n^2 - n - 2}.$

13. $\sum_{n=0}^{\infty} \frac{16}{n^2 + 4n + 3}.$

14. $\sum_{n=0}^{\infty} \frac{36}{n^2 + 7n + 10}.$

15. $\sum_{n=10}^{\infty} \frac{30}{n^2 - 14n + 48}.$

16. $\sum_{n=9}^{\infty} \frac{54}{n^2 - 11n + 28}.$

17. $\sum_{n=9}^{\infty} \frac{36}{n^2 - 12n + 35}.$

18. $\sum_{n=8}^{\infty} \frac{72}{n^2 - 9n + 18}.$

19. $\sum_{n=8}^{\infty} \frac{12}{n^2 - 10n + 24}.$

20. $\sum_{n=7}^{\infty} \frac{18}{n^2 - 7n + 10}.$

21. $\sum_{n=7}^{\infty} \frac{60}{n^2 - 8n + 15}.$

22. $\sum_{n=6}^{\infty} \frac{36}{n^2 - 5n + 4}.$

23. $\sum_{n=6}^{\infty} \frac{48}{n^2 - 6n + 8}.$

24. $\sum_{n=3}^{\infty} \frac{54}{n^2 + n - 2}.$

25. $\sum_{n=5}^{\infty} \frac{6}{n^2 - 4n + 3}.$

Задача 2. Найти сумму ряда.

1. $\sum_{n=3}^{\infty} \frac{4 - 5n}{n(n-1)(n-2)}.$

15. $\sum_{n=3}^{\infty} \frac{8n - 10}{(n-1)(n+1)(n-2)}.$

$$2. \sum_{n=1}^{\infty} \frac{n+6}{n(n+3)(n+2)}.$$

$$3. \sum_{n=1}^{\infty} \frac{5n+3}{n(n+1)(n+3)}.$$

$$4. \sum_{n=4}^{\infty} \frac{4n-2}{(n^2-1)(n-2)}.$$

$$5. \sum_{n=1}^{\infty} \frac{1}{n(n+1)(n+3)}.$$

$$6. \sum_{n=3}^{\infty} \frac{3n-5}{n(n^2-1)}.$$

$$7. \sum_{n=1}^{\infty} \frac{1}{n(n+2)(n+3)}.$$

$$8. \sum_{n=3}^{\infty} \frac{1}{n(n^2-4)}.$$

$$9. \sum_{n=1}^{\infty} \frac{3n-2}{n(n+1)(n+2)}.$$

$$10. \sum_{n=3}^{\infty} \frac{n+2}{n(n-1)(n-2)}.$$

$$11. \sum_{n=3}^{\infty} \frac{5n-2}{n(n-1)(n+2)}.$$

$$12. \sum_{n=1}^{\infty} \frac{2}{n(n+1)(n+2)}.$$

$$13. \sum_{n=1}^{\infty} \frac{3n+2}{n(n+1)(n+2)}.$$

$$14. \sum_{n=3}^{\infty} \frac{n+5}{(n+2)(n^2-1)}.$$

$$16. \sum_{n=3}^{\infty} \frac{3n-1}{n(n^2-1)}.$$

$$17. \sum_{n=3}^{\infty} \frac{n-4}{n(n-1)(n-2)}.$$

$$18. \sum_{n=1}^{\infty} \frac{5n+9}{n(n+1)(n+3)}.$$

$$19. \sum_{n=2}^{\infty} \frac{5n-2}{n(n-1)(n+2)}.$$

$$20. \sum_{n=1}^{\infty} \frac{n-1}{n(n+1)(n+2)}.$$

$$21. \sum_{n=1}^{\infty} \frac{3n+4}{n(n+1)(n+2)}.$$

$$22. \sum_{n=1}^{\infty} \frac{2-n}{n(n+1)(n+2)}.$$

Задача 3. Исследовать на сходимость ряд.

$$1. \sum_{n=1}^{\infty} \frac{\sin^2 n \sqrt{n}}{n \sqrt{n}}.$$

$$2. \sum_{n=1}^{\infty} \frac{\arctg^2 n}{n^3}.$$

$$3. \sum_{n=1}^{\infty} \frac{\arctg n^2}{n(n+1)(n+2)}.$$

$$4. \sum_{n=1}^{\infty} \frac{\ln n}{\sqrt[3]{n^7}}.$$

$$5. \sum_{n=1}^{\infty} \frac{3 - \sin n}{n - \ln n}.$$

$$6. \sum_{n=1}^{\infty} \frac{1 - \cos n}{n^3 + 2}.$$

$$7. \sum_{n=1}^{\infty} \frac{n(2 + \cos n\pi)}{2n^2 - 1}.$$

$$8. \sum_{n=2}^{\infty} \frac{3 + \sin n}{\sqrt[3]{n^3 - n}}.$$

$$9. \sum_{n=1}^{\infty} \frac{\sin^2 n}{n^2 + 1}.$$

$$10. \sum_{n=2}^{\infty} \frac{\ln \sqrt{n^2 + 3n}}{\sqrt{n^2 - n}}.$$

$$11. \sum_{n=1}^{\infty} \frac{1 + \cos n}{n^2 + 2}.$$

$$12. \sum_{n=1}^{\infty} \frac{n \cos^2 n}{n^3 + 5}.$$

$$13. \sum_{n=2}^{\infty} \frac{n \ln n}{n^2 - 3}.$$

$$14. \sum_{n=1}^{\infty} \frac{n^2 + 3}{n^3(2 + \cos n\pi)}.$$

$$15. \sum_{n=1}^{\infty} \frac{3 - \cos n}{\sqrt[4]{n^3}}.$$

$$16. \sum_{n=1}^{\infty} \frac{\ln n}{n^3 + n + 1}.$$

$$17. \sum_{n=1}^{\infty} \frac{\sin^2 n}{n^2}.$$

$$18. \sum_{n=1}^{\infty} \frac{\arctg^3 n}{n^4 + 3}.$$

$$19. \sum_{n=1}^{\infty} \frac{(2 + \cos n\pi)\sqrt{n}}{\sqrt[4]{n^7 + 5}}.$$

$$20. \sum_{n=1}^{\infty} \frac{1 - \sin n}{(n+1)(n+2)}.$$

$$21. \sum_{n=1}^{\infty} \frac{\sin^2 2^n}{n^2}.$$

$$22. \sum_{n=1}^{\infty} \frac{\ln n}{\sqrt{n^5 + n}}.$$

$$23. \sum_{n=2}^{\infty} \frac{1}{n^2 \ln n + \sqrt[3]{\ln^2 n}}.$$

$$24. \sum_{n=2}^{\infty} \frac{2 - \cos n}{\sqrt{n^2 - n}}.$$

$$25. \sum_{n=1}^{\infty} \frac{\arctg^2 n}{n(n+1)}.$$

Задача 4. Исследовать на сходимость ряд.

Указание: использовать таблицу эквивалентных функций

$$1. \sum_{n=1}^{\infty} \sqrt{n} \left(1 - \cos \frac{1}{n+1} \right).$$

$$2. \sum_{n=1}^{\infty} \frac{1}{n+4} \operatorname{tg} \frac{1}{\sqrt{n}}.$$

$$3. \sum_{n=1}^{\infty} \ln \frac{n^2 + 5}{n^2 + 4}.$$

$$4. \sum_{n=1}^{\infty} \frac{1}{\sqrt{n+4}} \sin \frac{1}{n+1}.$$

$$5. \sum_{n=2}^{\infty} \frac{1}{n-1} \arctg \frac{1}{\sqrt[3]{n-1}}.$$

$$6. \sum_{n=2}^{\infty} \ln \frac{n^2 + 3}{n^2 - n}.$$

$$7. \sum_{n=1}^{\infty} \left(e^{\frac{\sqrt{n}-1}{n^3}} - 1 \right).$$

$$8. \sum_{n=2}^{\infty} \sqrt{n} \arcsin \frac{n+1}{n^3-2}.$$

$$9. \sum_{n=1}^{\infty} n \left(e^{\frac{1}{\pi^2}} - 1 \right).$$

$$10. \sum_{n=1}^{\infty} \frac{1}{\sqrt[5]{n+1}} \sin \frac{1}{\sqrt{n}}.$$

$$11. \sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n}} \operatorname{arctg} \frac{\pi}{4\sqrt{n}}.$$

$$12. \sum_{n=2}^{\infty} \sqrt[3]{n} \cdot \operatorname{tg} \frac{n-1}{n^3-n}.$$

$$13. \sum_{n=2}^{\infty} \frac{1}{\sqrt[3]{n+5}} \sin \frac{1}{n-1}.$$

$$14. \sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n}+2} \operatorname{arctg} \frac{n+3}{n^2+5}.$$

$$15. \sum_{n=1}^{\infty} \frac{1}{\sqrt{n+3}} \left(e^{\frac{1}{\sqrt{\pi}}} - 1 \right).$$

$$16. \sum_{n=1}^{\infty} \ln \frac{n^2+1}{n^2-n+2}.$$

$$17. \sum_{n=1}^{\infty} \sqrt[3]{n} \cdot \operatorname{arctg} \frac{1}{n^3}.$$

$$18. \sum_{n=1}^{\infty} \frac{\sin \frac{\pi}{n}}{n+1}.$$

$$19. \sum_{n=3}^{\infty} n^3 \operatorname{tg}^5 \frac{\pi}{n}.$$

$$20. \sum_{n=2}^{\infty} \frac{e^{\frac{3}{n}} - 1}{(\sqrt[3]{n} - 1)}.$$

$$21. \sum_{n=1}^{\infty} \left(1 - \cos \frac{\pi}{n} \right).$$

$$22. \sum_{n=1}^{\infty} \sin \frac{\sqrt[3]{n}}{\sqrt{n^5+2}}.$$

$$23. \sum_{n=2}^{\infty} \left(e^{\frac{\sqrt{n}}{n^3-1}} - 1 \right).$$

$$24. \sum_{n=1}^{\infty} \sin \frac{2n+1}{n^2(n+1)^2}.$$

$$25. \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \sin \frac{2\pi}{2n+1}.$$

Задача 5. Исследовать на сходимость ряд.

$$1. \sum_{n=2}^{\infty} \frac{n+1}{2^n(n-1)!}.$$

$$2. \sum_{n=1}^{\infty} \frac{4^n}{(n)!}.$$

$$3. \sum_{n=1}^{\infty} \frac{2^{n+1}(n^3+1)}{(n+1)!}.$$

$$4. \sum_{n=1}^{\infty} \frac{10^n n!}{(2n)!}.$$

$$5. \sum_{n=1}^{\infty} \frac{(2n+2)!}{2^n(3n+5)}.$$

$$6. \sum_{n=1}^{\infty} \frac{n+5}{n!} \sin \frac{2}{3^n}.$$

$$7. \sum_{n=1}^{\infty} \frac{1}{n!} \operatorname{arctg} \frac{5}{n}.$$

$$8. \sum_{n=1}^{\infty} \frac{n^n}{3^n n!}.$$

$$9. \sum_{n=1}^{\infty} \frac{n!}{(2n)!} \operatorname{tg} \frac{1}{5^n}.$$

$$10. \sum_{n=1}^{\infty} \frac{6^n(n^2-1)}{n!}.$$

$$11. \sum_{n=1}^{\infty} \frac{4^n n^2}{(n+2)!}.$$

$$12. \sum_{n=1}^{\infty} \frac{n^n}{(n!)^2}.$$

$$13. \sum_{n=1}^{\infty} \frac{7^{2n}}{(2n-1)!}.$$

$$14. \sum_{n=1}^{\infty} \frac{4^n n!}{(3n)!}.$$

$$15. \sum_{n=1}^{\infty} \frac{1 \cdot 3 \cdot 5 \cdot \dots \cdot (2n-1)}{3^n(n+1)!}.$$

$$16. \sum_{n=1}^{\infty} \frac{n!}{n^{n-1}}.$$

$$17. \sum_{n=1}^{\infty} \frac{(n!)^2}{(3^n+1)(2n)!}.$$

$$18. \sum_{n=1}^{\infty} n! \sin \frac{\pi}{2^n}.$$

$$19. \sum_{n=1}^{\infty} \frac{(n+1)!}{n^n}.$$

$$20. \sum_{n=1}^{\infty} \frac{5^n \sqrt[3]{n^2}}{(n+1)!}.$$

$$21. \sum_{n=1}^{\infty} \frac{2^n n!}{n^n}.$$

$$22. \sum_{n=1}^{\infty} \frac{5^n(n+1)!}{(2n)!}.$$

$$23. \sum_{n=1}^{\infty} \frac{3^n}{4^n(n+2)!}.$$

$$24. \sum_{n=1}^{\infty} \frac{3 \cdot 5 \cdot 7 \cdot \dots \cdot (2n+1)}{2 \cdot 5 \cdot 8 \cdot \dots \cdot (3n-1)}.$$

$$25. \sum_{n=1}^{\infty} \frac{1 \cdot 4 \cdot 7 \cdot \dots \cdot (3n-2)}{7 \cdot 9 \cdot 11 \cdot \dots \cdot (2n+5)}.$$

Задача 6. Исследовать ряд на сходимость.

$$1. \sum_{n=1}^{\infty} \frac{1}{3^n} \left(\frac{n}{n+1} \right)^{-n^2}.$$

$$2. \sum_{n=1}^{\infty} n^4 \left(\frac{2n}{3n+5} \right)^n.$$

$$3. \sum_{n=1}^{\infty} \left(\frac{2n^2+1}{n^2+1} \right)^{n^2}.$$

$$4. \sum_{n=1}^{\infty} \left(1 + \frac{1}{n} \right)^{n^2} \cdot \frac{1}{4^n}.$$

$$5. \sum_{n=1}^{\infty} \left(\frac{2n+1}{3n-2} \right)^{n^2}.$$

$$6. \sum_{n=1}^{\infty} \left(\frac{2n+2}{3n+1} \right)^n \cdot n^3.$$

$$7. \sum_{n=1}^{\infty} \left(\frac{4n-3}{5n+1} \right)^{n^2}.$$

$$8. \sum_{n=1}^{\infty} \left(\frac{n}{10n+5} \right)^{n^2}.$$

$$9. \sum_{n=1}^{\infty} n \arcsin^n \frac{\pi}{4n}.$$

$$10. \sum_{n=1}^{\infty} \left(\frac{n+2}{3n-1} \right)^{n^2}.$$

$$11. \sum_{n=1}^{\infty} \left(\frac{n-1}{n} \right)^n \frac{n}{5^n}.$$

$$12. \sum_{n=1}^{\infty} \left(\frac{2n+3}{n+1} \right)^{n^2}.$$

$$13. \sum_{n=1}^{\infty} n^2 \left(\frac{3n+2}{4n-1} \right)^n.$$

$$14. \sum_{n=2}^{\infty} \left(\frac{n+1}{2n-3} \right)^{n^2}.$$

$$15. \sum_{n=1}^{\infty} \left(\frac{n}{3n+1} \right)^{2n+1}.$$

$$16. \sum_{n=1}^{\infty} \left(\frac{2n-1}{3n+1} \right)^{\frac{n}{2}}.$$

$$17. \sum_{n=1}^{\infty} \frac{2^{n+1}}{n^n}.$$

$$18. \sum_{n=1}^{\infty} n^2 \sin^n \frac{\pi}{2n}.$$

$$19. \sum_{n=2}^{\infty} \frac{n^3}{(\ln n)^n}.$$

$$20. \sum_{n=1}^{\infty} \left(\frac{n}{3n-1} \right)^{n^3}.$$

$$21. \sum_{n=1}^{\infty} n^3 \arctg^n \frac{\pi}{3n}.$$

$$22. \sum_{n=1}^{\infty} \frac{3^n n^5}{(2n+1)^n}.$$

$$23. \sum_{n=1}^{\infty} 2^{n-1} e^{-n}.$$

$$24. \sum_{n=1}^{\infty} n \left(\frac{3n-1}{4n+2} \right)^{2n}.$$

$$25. \sum_{n=1}^{\infty} \left(\frac{2n}{4n+3} \right)^{n^2}.$$

Задача 7. Исследовать на сходимость ряд.

Указание: см. пример в лекции.

$$1. \sum_{n=2}^{\infty} \frac{1}{n \ln^2(3n+1)}.$$

$$2. \sum_{n=1}^{\infty} \frac{1}{n \ln^2(2n+1)}.$$

$$3. \sum_{n=1}^{\infty} \frac{1}{(2n+3) \ln^2(2n+1)}.$$

$$4. \sum_{n=3}^{\infty} \frac{1}{(3n-5) \ln^2(4n-7)}.$$

$$5. \sum_{n=1}^{\infty} \frac{1}{(3n+4) \ln^2(5n+2)}.$$

$$6. \sum_{n=1}^{\infty} \frac{1}{(2n+1) \ln^2(n\sqrt{5}+2)}.$$

$$7. \sum_{n=1}^{\infty} \frac{1}{(n\sqrt{2}+1) \ln^2(n\sqrt{3}+1)}.$$

$$8. \sum_{n=5}^{\infty} \frac{1}{(n-2) \ln(n-3)}.$$

$$9. \sum_{n=1}^{\infty} \frac{1}{(2n-1)\ln(2n)}.$$

$$10. \sum_{n=1}^{\infty} \frac{1}{(n+1)\ln(2n)}.$$

$$11. \sum_{n=2}^{\infty} \frac{1}{(3n-1)\ln n}.$$

$$12. \sum_{n=2}^{\infty} \frac{1}{(2n-1)\ln(n+1)}.$$

$$13. \sum_{n=2}^{\infty} \frac{1}{(2n-3)\ln(3n+1)}.$$

$$14. \sum_{n=1}^{\infty} \frac{1}{(n+2)\ln^2 n}.$$

$$15. \sum_{n=2}^{\infty} \frac{1}{(n+3)\ln^2(2n)}.$$

$$16. \sum_{n=2}^{\infty} \frac{1}{(2n+3)\ln^2(n+1)}.$$

$$17. \sum_{n=3}^{\infty} \frac{1}{n\ln(n-1)}.$$

$$18. \sum_{n=2}^{\infty} \frac{1}{2n\sqrt{\ln(3n-1)}}.$$

$$19. \sum_{n=5}^{\infty} \frac{1}{(n-2)\sqrt{\ln(n-3)}}.$$

$$20. \sum_{n=4}^{\infty} \frac{1}{(3n-1)\sqrt{\ln(n-2)}}.$$

$$21. \sum_{n=2}^{\infty} \frac{1}{(n+5)\ln^2(n+1)}.$$

$$22. \sum_{n=2}^{\infty} \frac{1}{(n+3)\ln^2(n+7)}.$$

$$23. \sum_{n=2}^{\infty} \frac{n^2}{(n^3+1)\ln n}.$$

$$24. \sum_{n=3}^{\infty} \frac{n}{(n^2-3)\ln^2 n}.$$

$$25. \sum_{n=4}^{\infty} \frac{1}{(n-3)\ln^2(n/2)}.$$

Задача 8. Исследовать на сходимость ряд.

$$1. \sum_{n=1}^{\infty} (-1)^{n+1} \frac{2n+1}{n(n+1)}.$$

$$2. \sum_{n=1}^{\infty} (-1)^{n+1} \left(\frac{n}{2n+1} \right)^n.$$

$$3. \sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{\ln(n+1)}.$$

$$4. \sum_{n=3}^{\infty} \frac{(-1)^n}{n\ln n(\ln \ln n)}.$$

$$5. \sum_{n=1}^{\infty} (-1)^n \frac{2n^2}{n^4 - n^2 + 1}.$$

$$6. \sum_{n=3}^{\infty} \frac{(-1)^n}{(n+1)\ln n}.$$

$$7. \sum_{n=3}^{\infty} \frac{(-1)^n}{n\ln(n+1)}.$$

$$8. \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^4 \sqrt{2n+3}}.$$

$$9. \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{3n+1}} \sin \frac{\pi}{2\sqrt{n}}.$$

$$10. \sum_{n=1}^{\infty} (-1)^n \left(\frac{3n-1}{n} \right)^n.$$

$$11. \sum_{n=1}^{\infty} \frac{\sin n}{n!}.$$

$$12. \sum_{n=3}^{\infty} \frac{(-1)^n}{n\ln(2n)}.$$

$$13. \sum_{n=1}^{\infty} (-1)^n \operatorname{tg} \frac{1}{n}.$$

$$14. \sum_{n=1}^{\infty} \frac{\cos n}{n^2}.$$

$$15. \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2^{2n}(n+1)}.$$

$$16. \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[3]{3n} \cos(\pi/3n)}.$$

$$\begin{aligned}
17. & \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{(3/2)^n (n+1)}. \\
18. & \sum_{n=1}^{\infty} (-1)^n \frac{2n-1}{3n}. \\
19. & \sum_{n=1}^{\infty} (-1)^n \frac{(n+3)!}{2^n}. \\
20. & \sum_{n=1}^{\infty} (-1)^n \frac{n+1}{\sqrt{n^3}}. \\
21. & \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{5n-1}} \operatorname{tg} \frac{\pi}{4\sqrt{n}}.
\end{aligned}$$

$$\begin{aligned}
22. & \sum_{n=0}^{\infty} \frac{(-1)^n}{2^{2n+1} (2n+1)}. \\
23. & \sum_{n=1}^{\infty} (-1)^n \frac{\sin(n\sqrt{n})}{n\sqrt{n}}. \\
24. & \sum_{n=1}^{\infty} \frac{(-1)^n}{n + \cos(2/\sqrt{n+4})}. \\
25. & \sum_{n=1}^{\infty} (-1)^n \sin \frac{\pi}{2^n}.
\end{aligned}$$

Задача 9. Вычислить сумму ряда с точностью α .

$$\begin{aligned}
1. & \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{3n^2}, \alpha = 0,01. \\
2. & \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n!}, \alpha = 0,01. \\
3. & \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(2n)^3}, \alpha = 0,001. \\
4. & \sum_{n=1}^{\infty} \frac{(-1)^n}{n!(2n+1)}, \alpha = 0,001. \\
5. & \sum_{n=1}^{\infty} (-1)^n \frac{2n+1}{n^3(n+1)}, \alpha = 0,01. \\
6. & \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n+1)!}, \alpha = 0,0001. \\
7. & \sum_{n=1}^{\infty} (-1)^n \frac{n}{2^n}, \alpha = 0,1. \\
8. & \sum_{n=1}^{\infty} (-1)^n \frac{n^2}{3^n}, \alpha = 0,1. \\
9. & \sum_{n=1}^{\infty} \frac{(-1)^n n}{(2n-1)^2 (2n+1)^2}, \alpha = 0,001. \\
10. & \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!!}, \alpha = 0,0001. \\
11. & \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n)!!}, \alpha = 0,001. \\
12. & \sum_{n=0}^{\infty} \left(-\frac{2}{5}\right)^n, \alpha = 0,01. \\
13. & \sum_{n=1}^{\infty} (-1)^n \frac{n}{7^n}, \alpha = 0,0001.
\end{aligned}$$

$$\begin{aligned}
14. & \sum_{n=0}^{\infty} \left(-\frac{2}{3}\right)^n, \alpha = 0,1. \\
15. & \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n)!}, \alpha = 0,001. \\
16. & \sum_{n=0}^{\infty} \frac{(-1)^n}{3n!}, \alpha = 0,01. \\
17. & \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n)! 2n}, \alpha = 0,00001. \\
18. & \sum_{n=1}^{\infty} (-1)^n \frac{2n+1}{(2n)! n!}, \alpha = 0,001. \\
19. & \sum_{n=1}^{\infty} \frac{(-1)^n}{2^n n!}, \alpha = 0,001. \\
20. & \sum_{n=1}^{\infty} \frac{(-1)^n}{3^n n!}, \alpha = 0,0001. \\
21. & \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n)! n!}, \alpha = 0,00001. \\
22. & \sum_{n=0}^{\infty} \frac{(-1)^n}{3^n (n+1)}, \alpha = 0,001. \\
23. & \sum_{n=0}^{\infty} \frac{(-1)^n}{4^n (2n+1)}, \alpha = 0,001. \\
24. & \sum_{n=1}^{\infty} \frac{(-1)^n}{n^3}, \alpha = 0,01. \\
25. & \sum_{n=0}^{\infty} (-1)^n \frac{2^n}{(n+1)^n}, \alpha = 0,001.
\end{aligned}$$

Задача 10. Найти область сходимости ряда.

$$1. \sum_{n=1}^{\infty} \frac{(n-2)^3}{2n+3} (x+3)^{2n}.$$

$$2. \sum_{n=1}^{\infty} \frac{(-1)^n}{(n+1)5^n} (x-3)^n.$$

$$3. \sum_{n=1}^{\infty} \frac{(x-1)^{2n}}{n9^n}.$$

$$4. \sum_{n=1}^{\infty} \frac{(-1)^n (n+1)}{(n+3)^2 2^{n-1}} (x+7)^n.$$

$$5. \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n} (x-2)^{2n}.$$

$$6. \sum_{n=1}^{\infty} \frac{(n-5)^{2n+1}}{3n+8}.$$

$$7. \sum_{n=1}^{\infty} \frac{(-1)^n}{(n+3)\ln(n+3)} (x+6)^n.$$

$$8. \sum_{n=1}^{\infty} \frac{(x-6)^n}{(n+2)3^n}.$$

$$9. \sum_{n=1}^{\infty} \frac{(x+5)^{2n-1}}{(2n-1)4^n}.$$

$$10. \sum_{n=1}^{\infty} \frac{(x-7)^{2n-1}}{(2n^2-5n)4^n}.$$

$$11. \sum_{n=1}^{\infty} \frac{(x-2)^n}{(3n+1)2^n}.$$

$$12. \sum_{n=1}^{\infty} \frac{3n}{(5n-8)^3} (x-2)^{3n}.$$

$$13. \sum_{n=1}^{\infty} \frac{(x+5)^n}{3^n}.$$

$$14. \sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2+1} (x-2)^n.$$

$$15. \sum_{n=1}^{\infty} \frac{(-1)^n}{(3n+1)3^n} (x+6)^n.$$

$$16. \sum_{n=1}^{\infty} \frac{n+1}{(3n+1)^3} (x-4)^{2n}.$$

$$17. \sum_{n=1}^{\infty} \frac{(x-6)^n}{(n+3)2^n}.$$

$$18. \sum_{n=1}^{\infty} \frac{n^5}{(n+1)!} (x+5)^{2n+1}.$$

$$19. \sum_{n=1}^{\infty} \frac{3n-2}{(n+1)^2 2^n} (x-3)^n.$$

$$20. \sum_{n=1}^{\infty} \frac{(x-5)^n}{(n+4)\ln(n+4)}.$$

$$21. \sum_{n=1}^{\infty} \frac{(-1)^n}{(4n+1)3^n} (x+4)^n.$$

$$22. \sum_{n=1}^{\infty} \frac{n^2}{(n+2)!} (x+1)^{2n-1}.$$

$$23. \sum_{n=1}^{\infty} \frac{(-1)^n}{(3n-1)2^n} (x+3)^n.$$

$$24. \sum_{n=1}^{\infty} \frac{2n}{(3n+1)^3} (x-1)^{3n}.$$

$$25. \sum_{n=1}^{\infty} \frac{n^3}{(n+3)!} (x+4)^{2n+1}.$$

Задача 11. Найти область сходимости ряда.

$$1. \sum_{n=1}^{\infty} \frac{5^n}{n(x^2-6x+13)^n}.$$

$$2. \sum_{n=1}^{\infty} \frac{8^n}{n^2} \sin^{3n} x.$$

$$3. \sum_{n=1}^{\infty} \left(1 + \frac{1}{n}\right)^n 3^{\frac{n}{x-1}}.$$

$$4. \sum_{n=1}^{\infty} \frac{n+1}{3^n} (x^2-4x+6)^n.$$

$$5. \sum_{n=1}^{\infty} 8^n n^2 \sin^{3n} x.$$

$$6. \sum_{n=1}^{\infty} \frac{1}{n(n+1)} 2^{\frac{n}{4-x}}.$$

$$7. \sum_{n=1}^{\infty} \frac{4^n}{n(x^2 - 5x + 10)^n}.$$

$$8. \sum_{n=1}^{\infty} \frac{2^n}{\sqrt{n}} \sin^{2n}(2x).$$

$$9. \sum_{n=1}^{\infty} \frac{\ln^n(x+e)}{n+e}.$$

$$10. \sum_{n=1}^{\infty} \frac{(x^2 - 6x + 12)^n}{4^n(n^2 + 1)}.$$

$$11. \sum_{n=1}^{\infty} \frac{3^n}{n} \operatorname{tg}^{2n} x.$$

$$12. \sum_{n=1}^{\infty} n e^{-\frac{n}{\cos x}}.$$

$$13. \sum_{n=1}^{\infty} \frac{3^n}{n^2(x^2 + 2)^n}.$$

$$14. \sum_{n=1}^{\infty} \frac{2^n}{n^4} \sin^4(3x).$$

$$15. \sum_{n=1}^{\infty} \frac{1}{n} 4^{\frac{n}{x-2}}.$$

$$16. \sum_{n=1}^{\infty} \frac{(x^2 - 5x + 11)^n}{5^n(n^2 + 5)}.$$

$$17. \sum_{n=1}^{\infty} \frac{4^n}{n^2} \sin^{2n} x.$$

$$18. \sum_{n=1}^{\infty} \frac{\ln^n x}{2^n n^2}.$$

$$19. \sum_{n=1}^{\infty} \frac{3^n}{n(x^2 - 2x + 3)^n}.$$

$$20. \sum_{n=1}^{\infty} \frac{1}{n^3} \operatorname{tg}^n(2x).$$

$$21. \sum_{n=1}^{\infty} n e^{-n \sin x}.$$

$$22. \sum_{n=1}^{\infty} \frac{(x^2 + 1)^n}{2^n(n+1)}.$$

$$23. \sum_{n=1}^{\infty} \frac{1}{n^2} \operatorname{tg}^n x.$$

$$24. \sum_{n=1}^{\infty} n 5^{\frac{n}{3-x}}.$$

$$25. \sum_{n=1}^{\infty} \frac{2^n}{n^2(x^2 - 4x + 5)^n}.$$

Задача 12. Найти сумму ряда.

Указание: см. пример в лекции

$$1. \sum_{n=1}^{\infty} \frac{x^{n-1}}{n(n-1)}.$$

$$2. \sum_{n=1}^{\infty} \frac{5^n}{(n+1)x^n}.$$

$$3. \sum_{n=0}^{\infty} \frac{(1-x^2)^n}{n+1}.$$

$$4. \sum_{n=0}^{\infty} \frac{x^n}{(n+1)(n+2)}.$$

$$5. \sum_{n=1}^{\infty} \frac{2^{n-1}}{n x^{4n-4}}.$$

$$6. \sum_{n=1}^{\infty} \frac{(1-x^5)^{n-1}}{n}.$$

$$7. \sum_{n=1}^{\infty} \frac{x^n}{n(n+1)}.$$

$$8. \sum_{n=0}^{\infty} \frac{3^n}{(n+1)x^{3n}}.$$

$$9. \sum_{n=0}^{\infty} \frac{(1-x^4)^n}{n+1}.$$

$$10. \sum_{n=0}^{\infty} \frac{x^{n+1}}{(n+1)(n+2)}.$$

$$11. \sum_{n=1}^{\infty} \frac{5^{n-1}}{n x^{n-1}}.$$

$$12. \sum_{n=1}^{\infty} \frac{(1-x^2)^{n-1}}{n}.$$

$$13. \sum_{n=1}^{\infty} \frac{x^{n+2}}{n(n+1)}.$$

$$14. \sum_{n=0}^{\infty} \frac{1}{(n+1)x^{5n}}.$$

$$15. \sum_{n=0}^{\infty} \frac{\sin^n x}{n+1}.$$

$$16. \sum_{n=0}^{\infty} \frac{x^{n+3}}{(n+1)(n+2)}.$$

$$17. \sum_{n=1}^{\infty} \frac{3^{n-1}}{n x^{3n-3}}.$$

$$18. \sum_{n=1}^{\infty} \frac{(1-x^4)^{n-1}}{n}.$$

$$19. \sum_{n=1}^{\infty} \frac{x^{n+3}}{n(n+1)}.$$

$$20. \sum_{n=0}^{\infty} \frac{4^n}{(n+1)x^{2n}}.$$

$$21. \sum_{n=0}^{\infty} \frac{(1-x^3)^n}{n+1}.$$

$$22. \sum_{n=0}^{\infty} \frac{x^{n+4}}{(n+1)(n+2)}.$$

$$23. \sum_{n=1}^{\infty} \frac{1}{nx^{5n-5}}.$$

$$25. \sum_{n=1}^{\infty} \frac{x^{n+5}}{n(n+1)}.$$

$$24. \sum_{n=0}^{\infty} \frac{\cos^n x}{n+1}.$$

Задача 13. Найти сумму ряда.

Указание: см. пример в лекции

$$1. \sum_{n=1}^{\infty} (n+5)x^{n-1}.$$

$$10. \sum_{n=0}^{\infty} (n+1)x^{6n}.$$

$$19. \sum_{n=2}^{\infty} (n+1)x^{n-2}.$$

$$2. \sum_{n=0}^{\infty} (n+5)x^{2n}.$$

$$11. \sum_{n=2}^{\infty} nx^{n-2}.$$

$$20. \sum_{n=0}^{\infty} (n+1)x^{2n+2}.$$

$$3. \sum_{n=1}^{\infty} (n+4)x^{n-1}.$$

$$12. \sum_{n=1}^{\infty} nx^{6n}.$$

$$21. \sum_{n=0}^{\infty} (n+1)x^{2n}.$$

$$4. \sum_{n=0}^{\infty} (n+4)x^{3n}.$$

$$13. \sum_{n=2}^{\infty} (n+4)x^{n-2}.$$

$$22. \sum_{n=3}^{\infty} (n+1)x^{n-3}.$$

$$5. \sum_{n=1}^{\infty} (n+3)x^{n-1}.$$

$$14. \sum_{n=1}^{\infty} nx^{5n}.$$

$$23. \sum_{n=0}^{\infty} (n+2)x^{3n}.$$

$$6. \sum_{n=0}^{\infty} (n+3)x^{4n}.$$

$$15. \sum_{n=2}^{\infty} (n+3)x^{n-2}.$$

$$24. \sum_{n=3}^{\infty} (n+2)x^{n-3}.$$

$$7. \sum_{n=1}^{\infty} (n+2)x^{n-1}.$$

$$16. \sum_{n=2}^{\infty} nx^{4n}.$$

$$25. \sum_{n=0}^{\infty} (n+3)x^{4n}.$$

$$8. \sum_{n=0}^{\infty} (n+2)x^{5n}.$$

$$17. \sum_{n=2}^{\infty} (n+2)x^{n-2}.$$

$$9. \sum_{n=1}^{\infty} (n+1)x^{n-1}.$$

$$18. \sum_{n=0}^{\infty} (n+1)x^{3n+3}.$$

Задача 14. Разложить функцию в ряд Тейлора по степеням x .

$$1. \frac{9}{20-x-x^2}.$$

$$9. (x-1)\sin 5x$$

$$18. \ln(1+2x-8x^2)$$

$$2. \frac{x^2}{\sqrt{4-5x}}.$$

$$10. \frac{ch3x-1}{x^2}.$$

$$19. 2x \sin^2\left(\frac{x}{2}\right) - x$$

$$3. \ln(1-x-6x^2)$$

$$11. \frac{6}{8+2x-x^2}.$$

$$20. (x-1)shx$$

$$4. 2x \cos^2\left(\frac{x}{2}\right) - x$$

$$12. \frac{1}{\sqrt[4]{16-3x}}.$$

$$21. \frac{5}{6+x-x^2}.$$

$$5. \frac{sh2x}{x} - 2.$$

$$13. \ln(1-x-12x^2)$$

$$22. x^3 \sqrt{27-2x}$$

$$6. \frac{7}{12+x-x^2}.$$

$$14. (3+e^{-x})^2$$

$$23. \ln(1+x-12x^2)$$

$$7. \frac{x}{\sqrt[3]{27-2x}}.$$

$$15. \frac{\arcsin x}{x} - 1.$$

$$24. \frac{\sin 3x}{x} - \cos 3x.$$

$$8. \ln(1+x-6x^2)$$

$$16. \frac{7}{12-x-x^2}.$$

$$25. \frac{\arctg x}{x}.$$

$$17. x^2 \sqrt{4-3x}$$

Задача 15. Вычислить интеграл с точностью до 0,001.

1. $\int_0^{0,1} e^{-6x^2} dx.$

2. $\int_0^{0,1} \sin(100x^2) dx.$

3. $\int_0^1 \cos x^2 dx.$

4. $\int_0^{0,5} \frac{dx}{\sqrt[4]{1+x^4}}.$

5. $\int_0^{0,1} \frac{1-e^{-2x}}{x} dx.$

6. $\int_0^1 \frac{\ln(1+x/5)}{x} dx.$

7. $\int_0^{1,5} \frac{dx}{\sqrt[3]{27+x^3}}.$

8. $\int_0^{0,2} e^{-3x^2} dx.$

9. $\int_0^{0,2} \sin(25x^2) dx.$

10. $\int_0^{0,5} \cos(4x^2) dx.$

11. $\int_0^1 \frac{dx}{\sqrt[4]{16+x^4}}.$

12. $\int_0^{0,2} \frac{1-e^{-x}}{x} dx.$

13. $\int_0^{0,4} \frac{\ln(1+x/2)}{x} dx.$

14. $\int_0^2 \frac{dx}{\sqrt[3]{64+x^3}}.$

15. $\int_0^{0,3} e^{-2x^2} dx.$

16. $\int_0^{0,4} \sin\left(\frac{5x}{2}\right)^2 dx.$

17. $\int_0^{0,2} \cos(25x^2) dx.$

18. $\int_0^{1,5} \frac{d}{\sqrt[4]{81+x^4}} x.$

19. $\int_0^{0,4} \frac{1-e^{-x/2}}{x} dx.$

20. $\int_0^{0,1} \frac{\ln(1+2x)}{x} dx.$

21. $\int_0^{2,5} \frac{dx}{\sqrt[3]{125+x^3}}.$

22. $\int_0^{0,4} e^{-3x^2/4} dx.$

23. $\int_0^{0,5} \sin(4x^2) dx.$

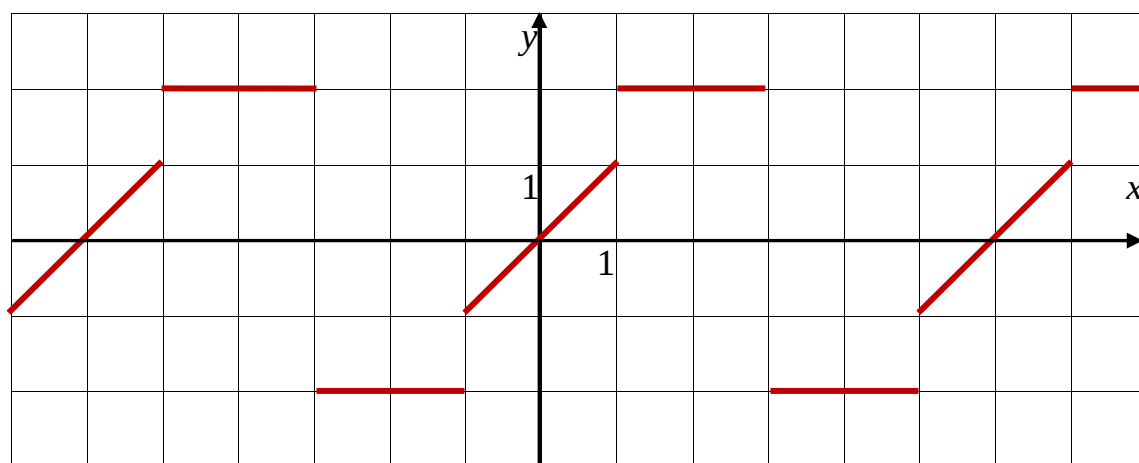
24. $\int_0^{0,4} \cos\left(\frac{5x}{2}\right)^2 dx.$

25. $\int_0^2 \frac{dx}{\sqrt[4]{256+x^4}}.$

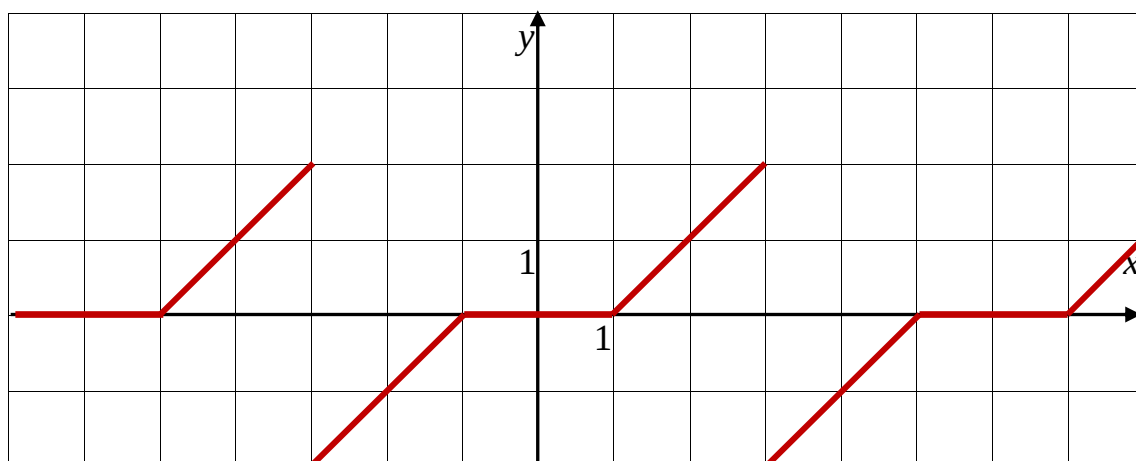
Задача 16

Разложить функцию, график которой изображен на чертеже, в ряд Фурье.

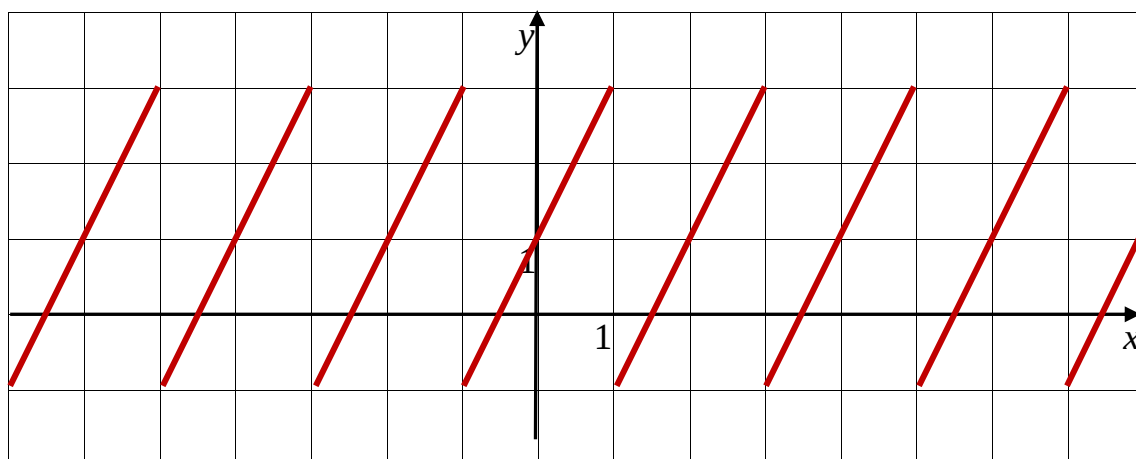
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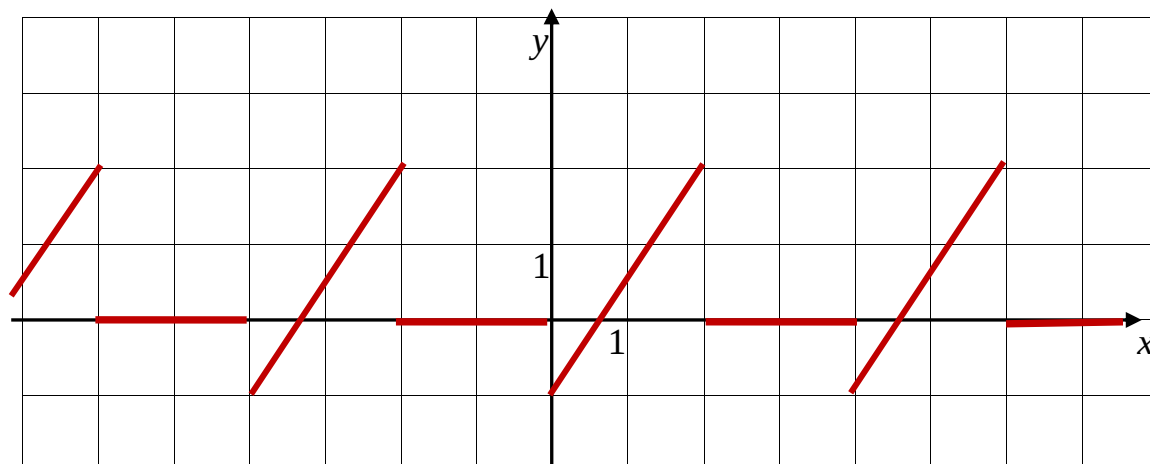
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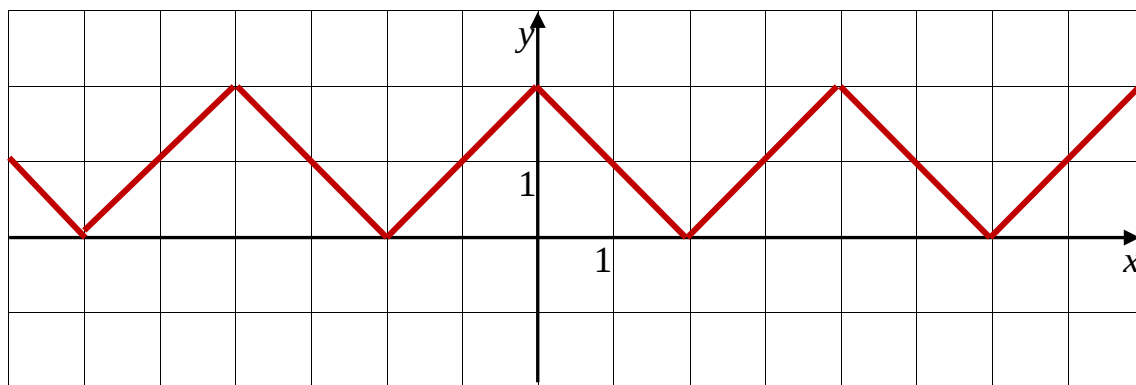
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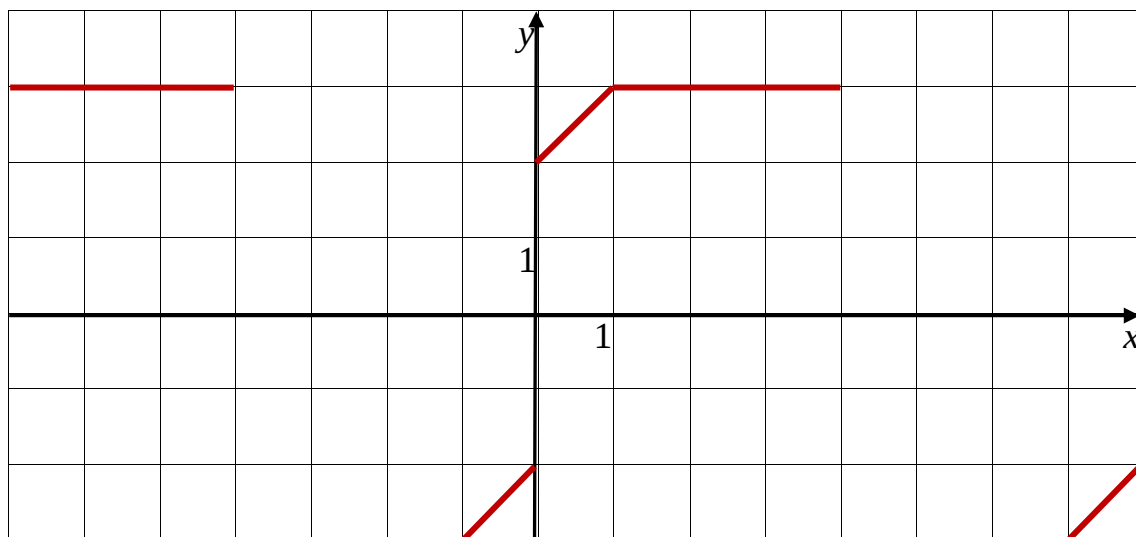
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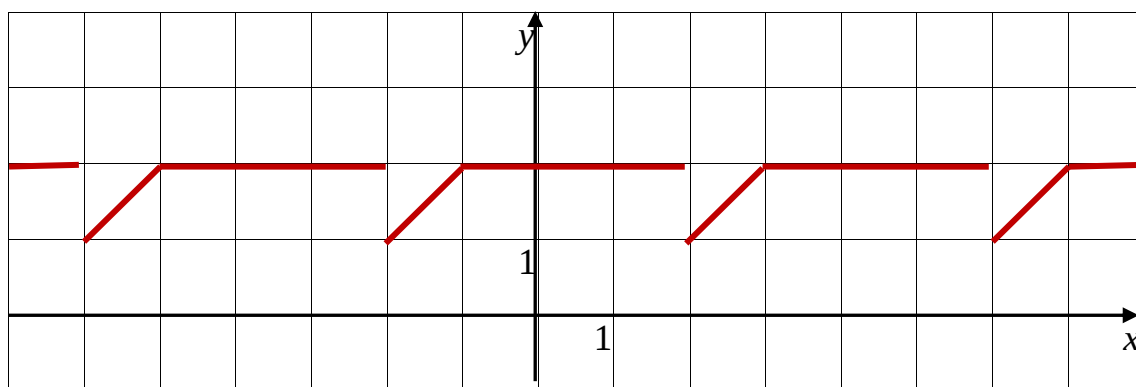
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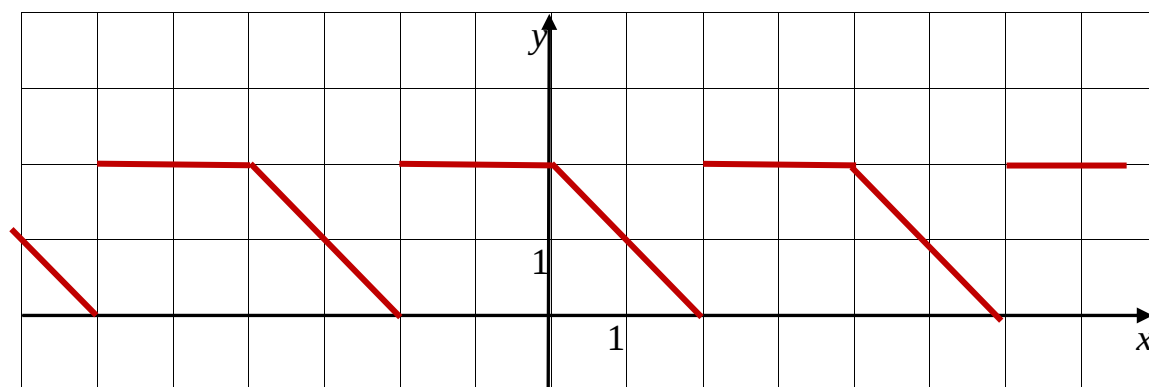
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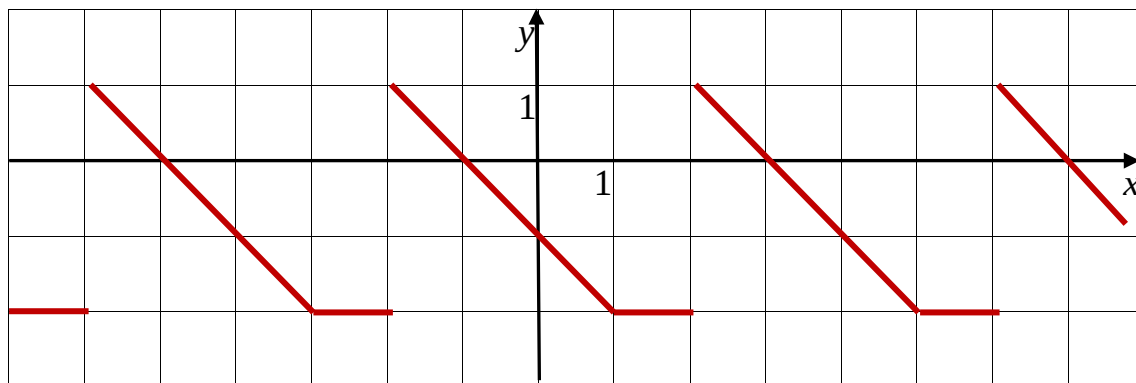
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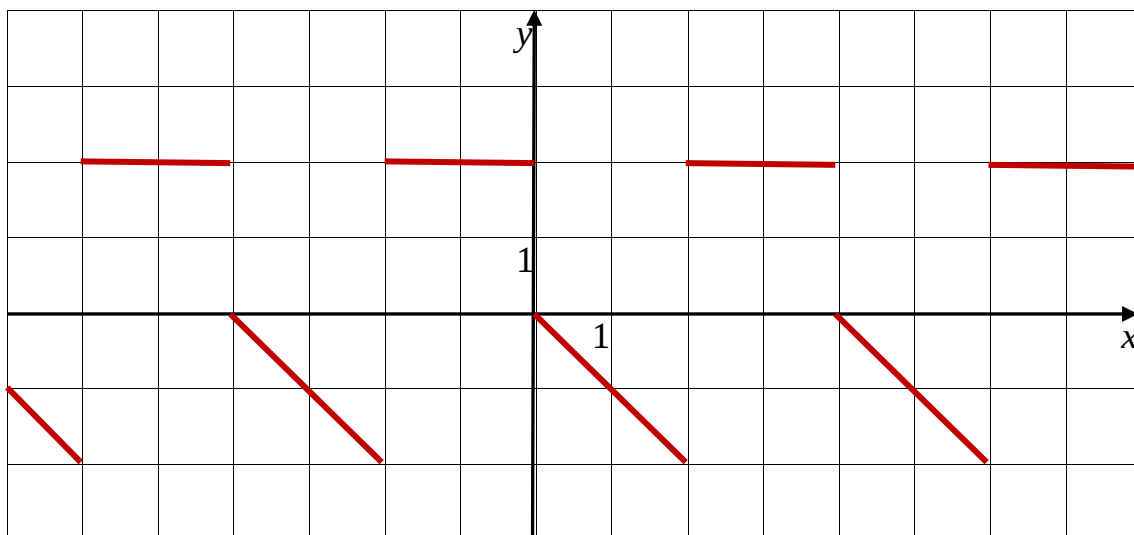
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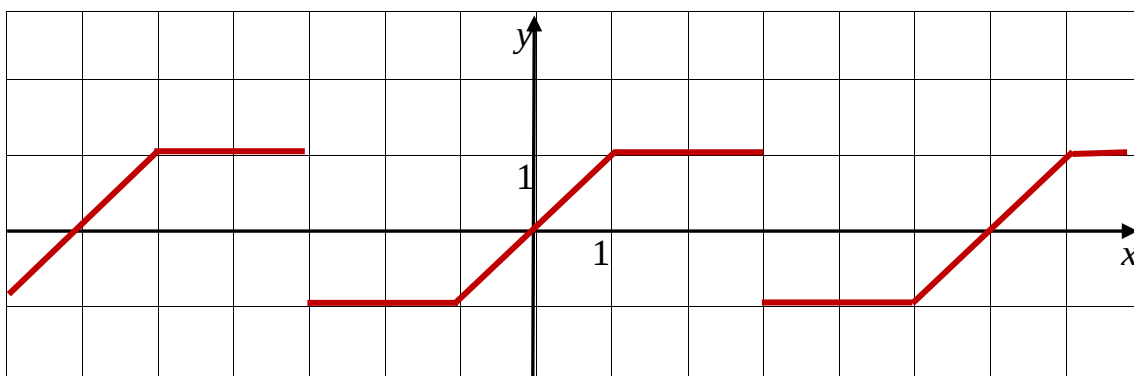
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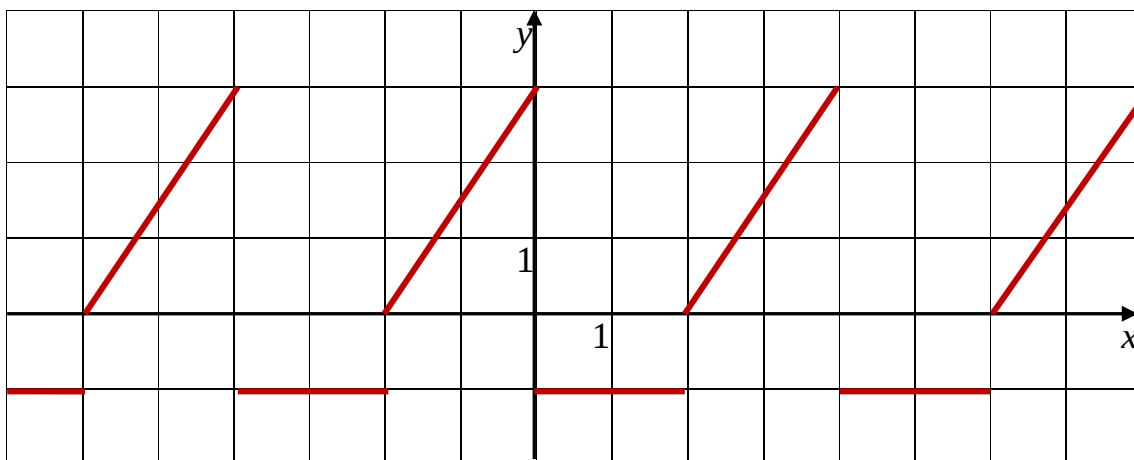
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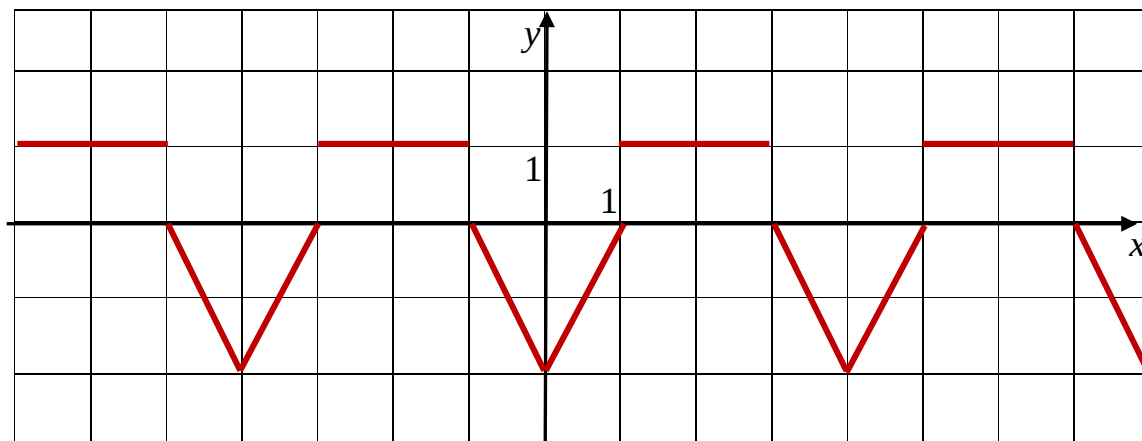
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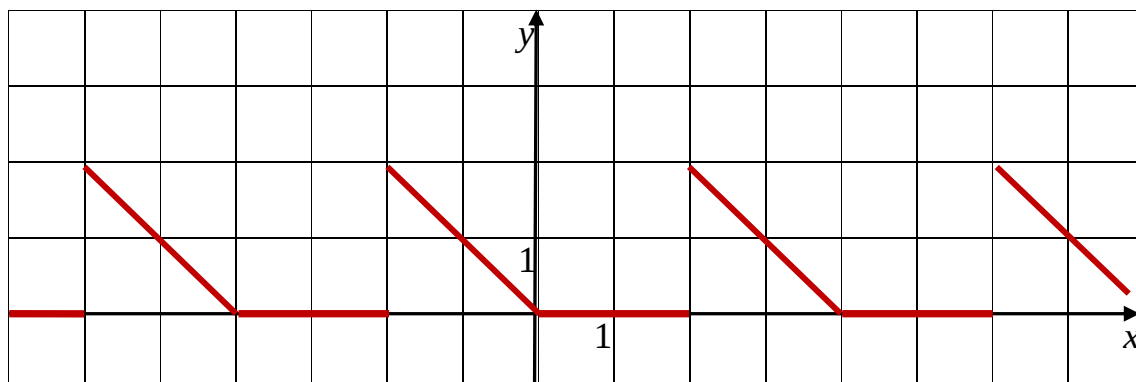
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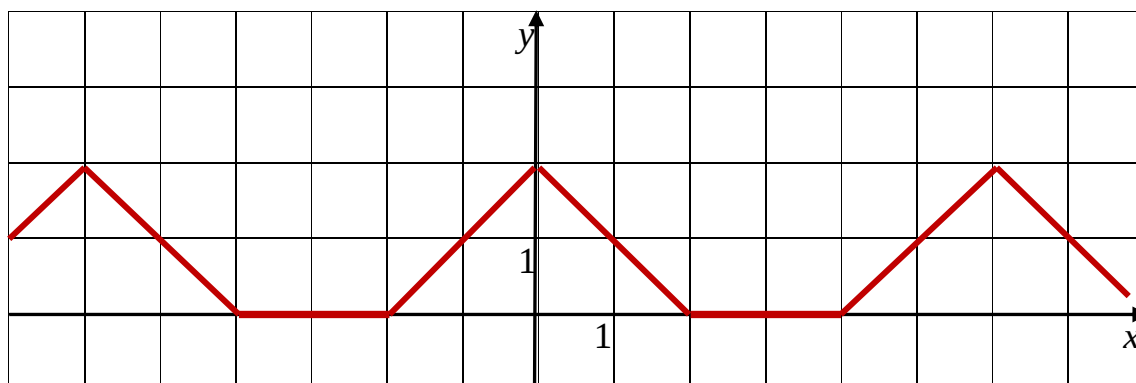
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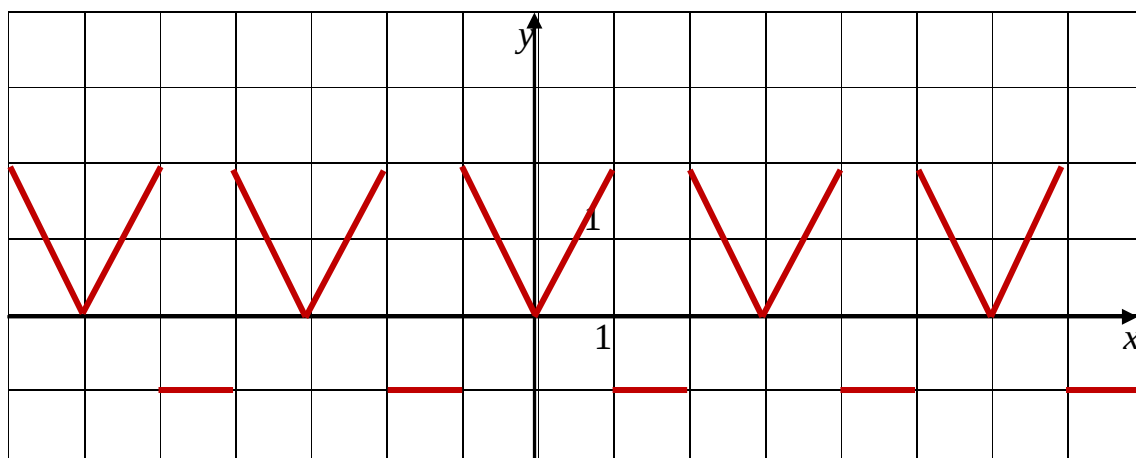
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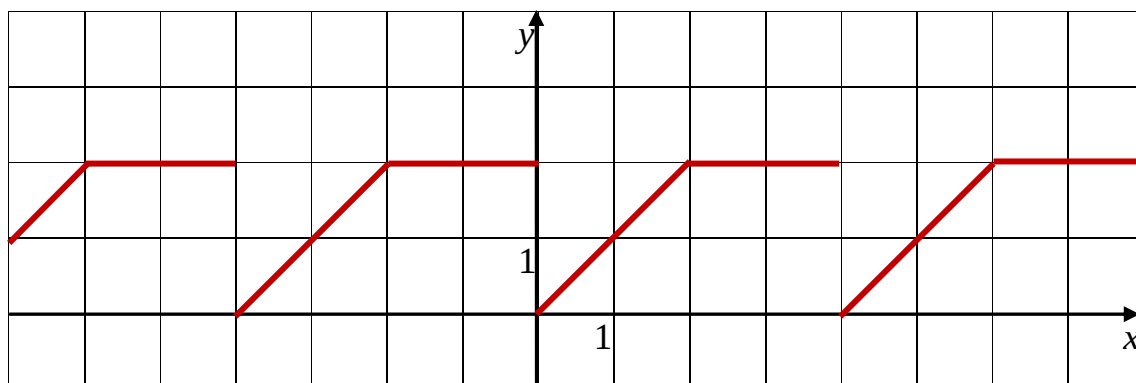
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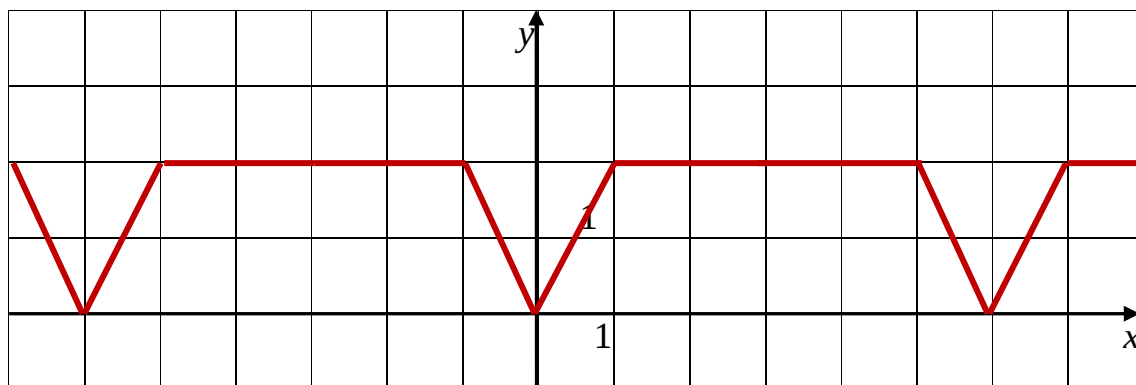
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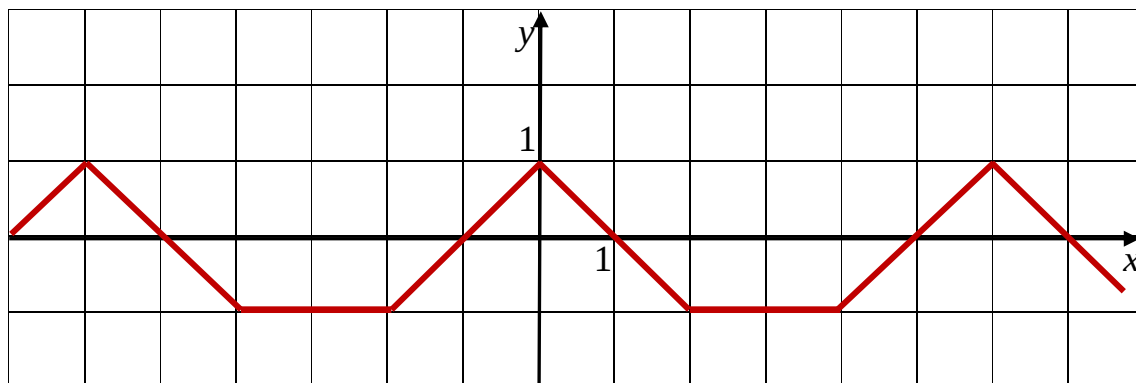
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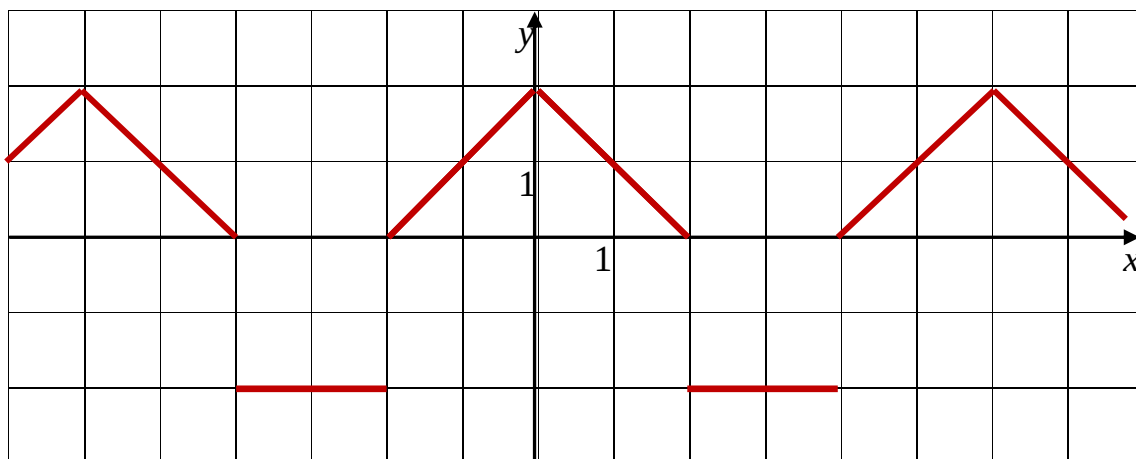
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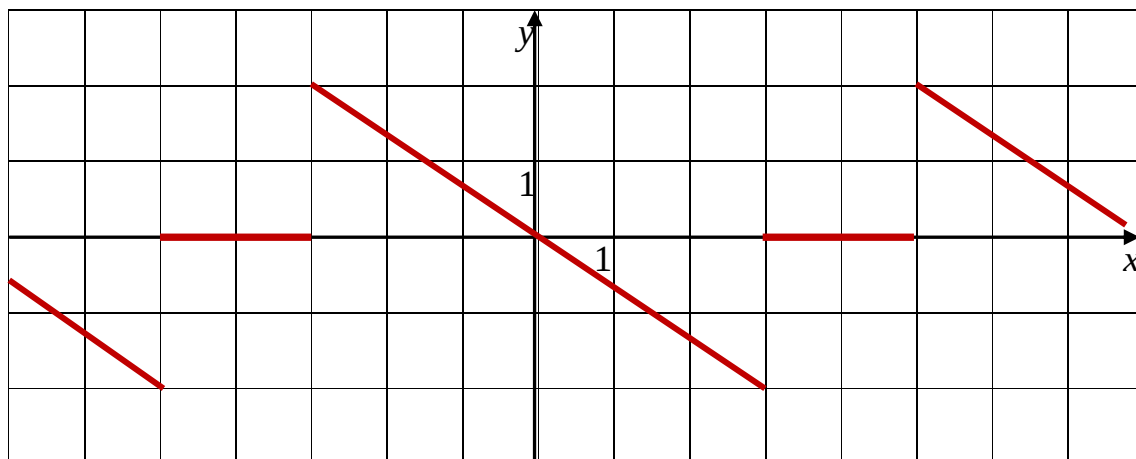
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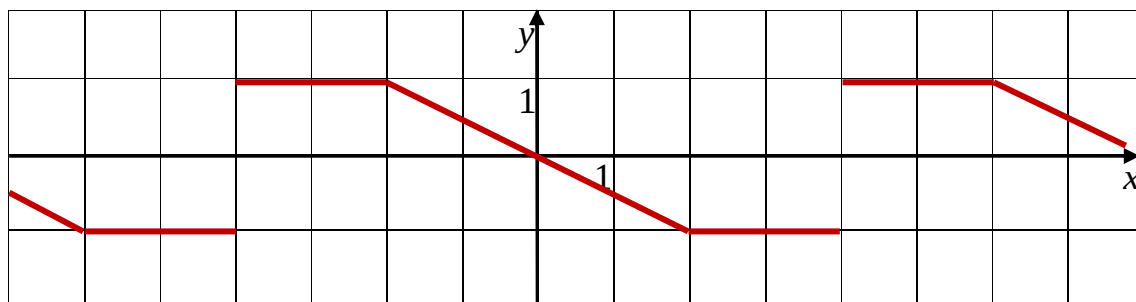
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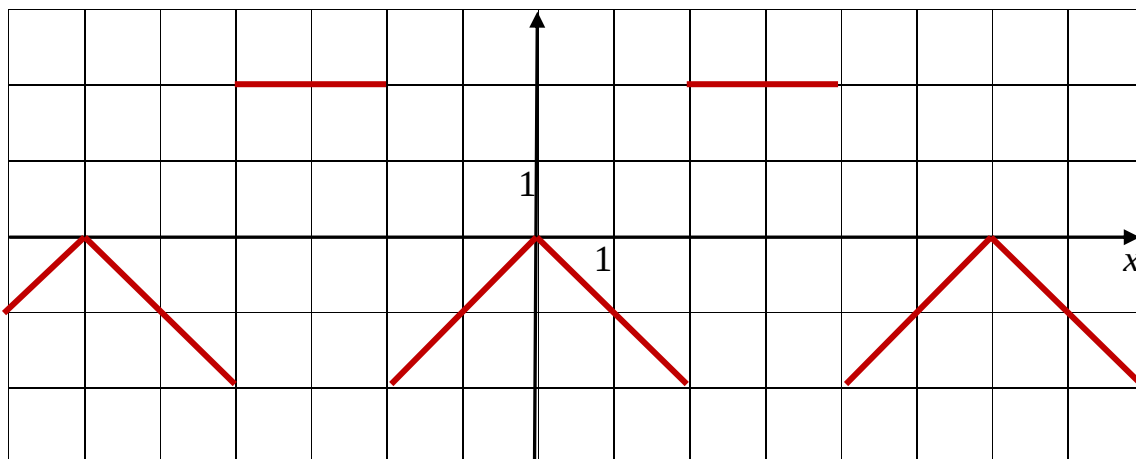
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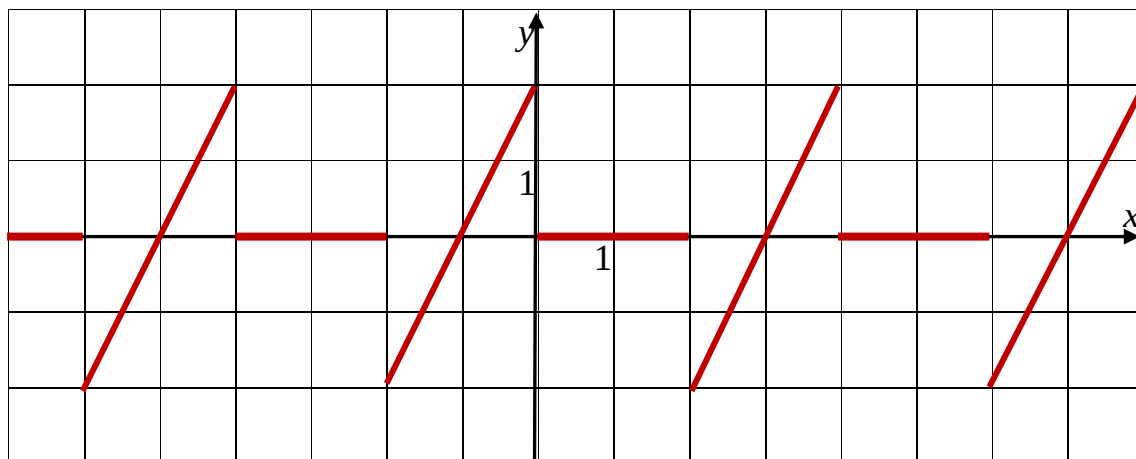
22.



23.



24.



25.

